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One contribution to a special feature 'A generation of network science' organized by Danica Vukadinovic-Greetham and Kristina Lerman.

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The transsortative structure of networks

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Network topologies can be highly non-trivial, due to the complex underlying behaviours that form them. While past research has shown that some processes on networks may be characterized by local statistics describing nodes and their neighbours, such as degree assortativity, these quantities fail to capture important sources of variation in network structure. We define a property called transsortativity that describes correlations among a node's neighbours. Transsortativity can be systematically varied, independently of the network's degree distribution and assortativity. Moreover, it can significantly impact the spread of contagions as well as the perceptions of neighbours, known as the majority illusion. Our work improves our ability to create and analyse more realistic models of complex networks.

1. Introduction

Networks serve as a substrate for the spread of contagion in social groups [1], propagation of information in online platforms [2] and cascading failures in the electrical power grid as well as in the financial sector [3–11]. Networks are frequently modelled using random graphs [12–15] that preserve certain statistical properties of real networks, such as degree distribution or degree assortativity [16], while removing other structure. These random graph models have been critical to understanding phenomena such as percolation, disease propagation, and ferromagnetism [3–5,8,9,17–19]. However, networks also exhibit substantial non-local structure as manifested by large numbers of connected triplets and bigger motifs [20], as well as degree [21] or attribute [22] correlations of two-hop, or even more distant, neighbours. This higher-order structure is necessary to explain effects such as the strong friendship paradox [23], where the majority of a node's neighbours have higher degree than the node itself [24].

We describe a method to measure, model and vary a higher-order network property that we call transsortativity. This property measures degree correlations among a node's neighbours (which are a two-hop distance away from each other). We illustrate how it can significantly alter network structure and network phenomena. Namely, we show that transsortativity amplifies the 'majority illusion' effect, where an unpopular idea may be perceived as popular by a large fraction of individuals, and also that it impacts the size and critical threshold for cascades in the Watts threshold model [1]. Our work therefore complements recent efforts to extend metrics of node degree correlations to more realistic situations, such as varying within-group assortative mixing [25] and long-range degree correlations [21,26–28]. We also show that transsortativity helps generalize overdispersion (monophily) in social networks [22,29], where the attributes of a node's neighbours display a larger variance than expected, and also describes the less familiar case of underdispersion.

Finally, we describe a rewiring procedure to systematically vary a network's transsortativity while keeping its degree distribution and assortativity fixed. Our examples demonstrate that transsortativity is an important tool in the statistical modelling of networks, and can be used with configuration models [12,13] and random rewiring [15] to create more realistic random networks.

2. Results

(a) Quantifying transsortativity in networks

Our analysis is motivated by the dK-series of probability distributions [30], which specifies the joint distribution of the degrees of connected subgraphs of d nodes. This provides a useful framework for characterizing network structure. The degree distribution of a network, $p(k)$, represents its *1K* structure. The joint degree distribution of pairs of adjacent nodes, $e(k, k')$, represents its *2K* structure. The Pearson correlation coefficient of the degrees of a node and of its neighbour is known as the *degree assortativity* [16]:

$$\begin{aligned} r_{2K} &= \frac{\mathrm{Cov}(k,k')}{\mathrm{Var}(k)} \\ &= \frac{\sum_{k,k'} kk'\left[e(k,k') - q(k)q(k')\right]}{\sum_k k^2 q(k) - \left[\sum_k kq(k)\right]^2}, \end{aligned} \tag{2.1}$$

where $q(k) = \sum_{k'} e(k, k') = kp(k)/\langle k\rangle$ is the degree distribution of a node that is adjacent to another, and $\mathrm{Cov}(k, k')$ and $\mathrm{Var}(k)$ are taken with respect to $q(k)$.

Now consider the neighbours of a degree-k node. Their degree distribution is $\nu(k'|k) = e(k, k')/q(k)$. In many real-world networks, given a pair of such neighbours i and j, one finds that their degrees k'_i and k'_j are correlated even if i and j are not themselves linked by an edge [23]. This two-hop degree correlation reflects the higher-order network structure, specifically the *3K* structure characterizing connected subgraphs with three nodes forming a 'wedge' or a 'triangle', as shown in figure 6.

Let $w(k'_i, k'_j|k)$ denote the joint degree distribution for those two neighbours of a degree-k node. This gives the probability that a degree-k node will have neighbours with degrees k'_i and k'_j. Note that this formulation tracks how many shared neighbours of degree k each pair of nodes i and j

has. We define the correlation coefficient of k'_i and k'_j, conditioned on k, as

$$r_{3K}(k) = \frac{\mathrm{Cov}(k'_i, k'_j|k)}{\mathrm{Var}(k'|k)}$$
$$= \frac{\sum_{k'_i,k'_j} k'_i k'_j \left[w(k'_i, k'_j|k) - \nu(k'_i|k)\nu(k'_j|k)\right]}{\sum_{k'}(k')^2\nu(k'|k) - [\sum_{k'}\nu(k'|k)]^2}, \tag{2.2}$$

where $\mathrm{Cov}(k'_i, k'_j|k)$ and $\mathrm{Var}(k'|k)$ are taken with respect to $\nu(k'|k)$. We refer to $r_{3K}(k)$ as *transsortativity*, because it measures correlations *across* neighbours rather than between a node and its neighbour. Transsortativity generalizes the notion of assortativity from immediate, or one-hop, neighbours to two-hop neighbours.

Values of transsortativity are bounded. To see why, consider the mean degree of a neighbour of a degree-k node, $\bar{k}' = \sum_i k'_i/k$. The variance of this quantity is

$$\mathrm{Var}(\bar{k}'|k) = \frac{1}{k^2}\left[\sum_{i=1}^{k}\mathrm{Var}(k'_i|k) + 2\sum_{i=1}^{k-1}\sum_{j=i+1}^{k}\mathrm{Cov}(k'_i, k'_j|k)\right]$$
$$= \frac{\mathrm{Var}(k'|k)}{k}[1 + (k-1)r_{3K}(k)]. \tag{2.3}$$

Non-negativity of the variance gives the lower bound:

$$r_{3K}(k) \geq -\frac{1}{k-1}. \tag{2.4}$$

Examples of transsortativity in real-world networks [31] are given in figure 1, showing that observed values of $r_{3K}(k)$ are large in cases ranging from a biological network of protein–protein interactions (Reactome), to co-authorship networks between physicists (ArXiv HepPh and GR), to hyperlink networks between webpages (Google), to friendship social networks (Facebook). Note that in most of these networks, transsortativity values are positive, implying assortative mixing between two-hop neighbours, regardless of the degree assortativity of immediate neighbours. Surprisingly, the Facebook social graph exhibits substantially negative transsortativity for low-degree nodes. This implies that low-degree nodes are connected to both low-degree and high-degree neighbours.

By averaging over all degrees in the network, we can calculate the mean transsortativity, analogous to equation (2.1):

$$\bar{r}_{3K} = \sum_{k=2}^{\infty} p(k) r_{3K}(k), \tag{2.5}$$

which, in turn, implies that

$$\bar{r}_{3K} \geq -\sum_{k=2}^{\infty} p(k)\frac{1}{k-1} \geq -\left\langle \frac{1}{k-1}\right\rangle. \tag{2.6}$$

Negative transsortativity is bounded by the mean of the inverse and is therefore typically small.

(b) Transsortativity rewiring algorithm

We use a rewiring algorithm [19] that preserves the degree distribution and degree assortativity (i.e. *1K* and *2K* structure), but can independently vary the transsortativity (*3K* structure). The algorithm is illustrated in figure 2. First, the algorithm chooses at random two nodes v_0 and w_0 of equal degree k_0. Then, it chooses at random one of the k_0 neighbours of v_0, denoted v_1, and one of the k_0 neighbours of w_0, denoted w_1. To decrease transsortativity (figure 2*a*), edges $\{v_0, v_1\}$ and $\{w_0, w_1\}$ are replaced with edges $\{v_0, w_1\}$ and $\{w_0, v_1\}$ if the edge swap makes v_0 and w_0 have more diverse neighbour degrees, i.e. smaller $r_{3K}(k_0)$. To increase transsortativity (figure 2*b*), the edges are swapped if this makes v_0 and w_0 have more similar neighbour degrees, i.e. larger

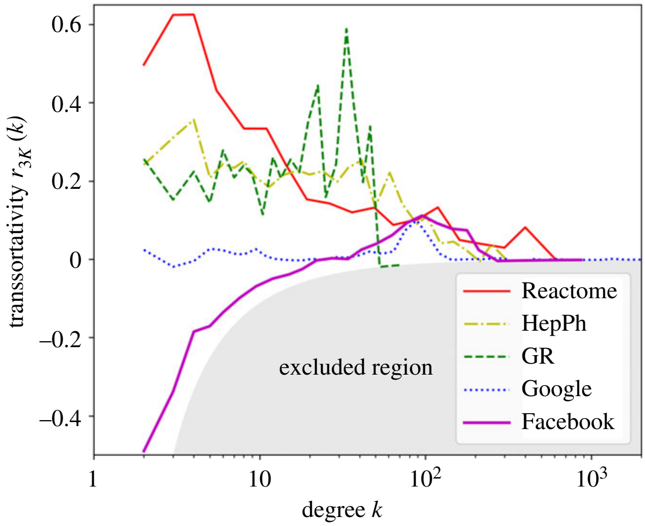

Figure 1. Transsortativity of networks from a variety of domains. The networks [31] include biological (Reactome), co-authorship (HepPh, GR), technological (Google) and social (Facebook). See appendix A for further examples. Grey region shows transsortativity values excluded by the theoretical lower bound (equation (2.4)). Data are aggregated using log-binning on degree k. (Online version in colour.)

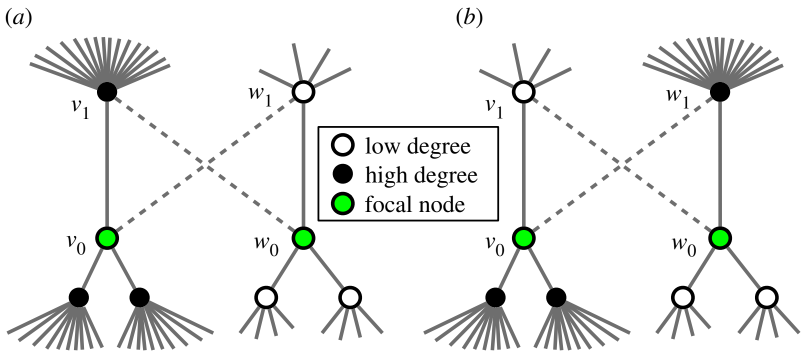

Figure 2. Transsortativity rewiring. The algorithm takes two nodes, v_0 and w_0, that have the same degree, and picks respective neighbours v_1 and w_1. (a) To reduce transsortativity, v_1 and w_1 swap edges (dashed lines) if this makes neighbour degrees become more diverse. (b) To increase transsortativity, v_1 and w_1 swap edges if this makes neighbour degrees become more similar. Since v_0 and w_0 have the same degree, the degree distribution and assortativity remain unchanged. (Online version in colour.)

$r_{3K}(k_0)$. Our implementation of the algorithm is partly annealed: we initially set the probability of rewiring in the undesirable direction to a finite value (0.5), then lower it over time to reduce undesirable edge swaps. The greedier rewiring strategy allows for achieving more extreme values of transsortativity, although in either case we can carefully control the final mean transsortativity value. We note that this may potentially result in non-ergodic dynamics, with subtle dependence on initial conditions. However, the good agreement between the theory and networks suggests that the risk of undesirable artefacts due to non-ergodicity is small.

We further illustrate the algorithm on Zachary's Karate club network [32], shown in figure 3. This network contains 34 members of a Karate club, with 78 social ties between them. The network is highly disassortative, with $r_{2K} = -0.476$. Before rewiring, the original Karate club

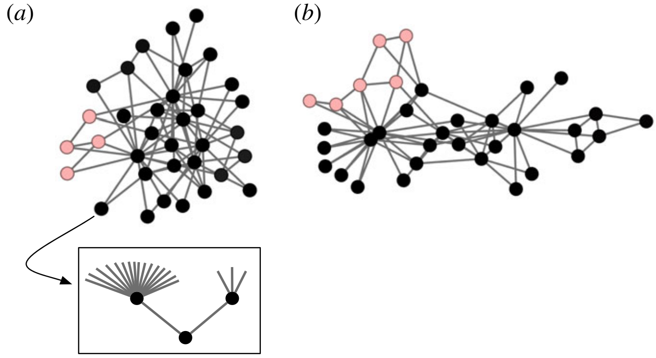

Figure 3. Transsortativity rewiring of Zachary's Karate Club network. (*a*) Negatively transsortative ($\bar{r}_{3K} = -0.40$), (*b*) original ($\bar{r}_{3K} = -0.098$) and (*c*) positively transsortative ($\bar{r}_{3K} = 0.46$) versions of the network. Lighter dots correspond to the giant vulnerable cluster (GVC) in the Watts model with threshold $\phi = 0.2$ (see cascade discussion in main text). Insets: examples of neighbour degree correlations within the negatively and positively transsortative networks. (Online version in colour.)

network has neutral transsortativity: $\bar{r}_{3K} = -0.098$ (figure 3*b*). Our rewiring algorithm can create networks with mean transsortativity ranging from $\bar{r}_{3K} = -0.40$ (figure 3*a*) to $\bar{r}_{3K} = 0.46$ (figure 3*c*). While the degree distribution and degree assortativity are identical in all cases, nodes in the negatively transsortative network have neighbours with widely varying degree, while nodes in the positively transsortative network have neighbours with similar degree (see figure insets), producing very different topologies.

(c) Transsortativity and network phenomena

(i) Majority illusion

We now consider the impact transsortativity has on network phenomena. First, we look at networks where nodes have particular attributes: examples might be gender, political affiliation or economic status. It has been shown that certain topologies produce a 'majority illusion' [33], where a significant fraction of nodes observe that a majority of their neighbours have a specific attribute, even when it is uncommon. Transsortativity can amplify the majority illusion. To understand why, consider a hypothetical social network where an individual's popularity correlates with an attribute such as happiness [34]. As a consequence, happier people would be more popular in this network and vice versa. Thus, even if only a small minority of individuals are happy, they would have a tendency to share many neighbours. These neighbours see a large fraction of friends that are happy, and a naive observer would conclude that most of his or her friends are happy.

The following straightforward analysis demonstrates this phenomenon explicitly. Consider a degree-k node with a binary attribute $x \in \{0, 1\}$, such as gender or political affiliation, and assume that $x = 0$ for a majority of nodes. Let $f(k)$ be the probability that a majority of its k neighbours have attribute value $x' = 1$. The overall probability of majority illusion is

$$P_{>\frac{1}{2}} = \sum_{k=1}^{k_{\max}} p(k) f(k). \tag{2.7}$$

Suppose that neighbour attributes simply arose as the outcomes of independent Bernoulli random trials with success probability denoted $\mu_x(k) = P(x' = 1|k)$. Then, since $f(k)$ is the probability of having more than $k/2$ such successes, it could be expressed using a binomial distribution and corresponding Gaussian approximation:

$$f(k) = \sum_{m=\lceil \frac{k+1}{2} \rceil}^{k} \binom{k}{m} \mu_x(k)^m [1-\mu_x(k)]^{k-m}$$

$$\approx 1 - \Phi\left[\frac{1-2\mu_x(k)}{2\sigma_x(k)}\right], \tag{2.8}$$

where Φ is the cumulative distribution function of the normal distribution, and $\sigma_x^2(k) = \mu_x(k)[1-\mu_x(k)]/k$ is the variance in the mean neighbour attribute value of a degree-k node.

However, in networks where node attributes are correlated with their degrees, transsortativity leads to correlations between attributes x'_i, x'_j of pairs of two-hop neighbours. Assuming the network is locally tree-like and no higher-order correlations exist, such as among connected subgraphs of four nodes (4K structure), it is sufficient to replace the expression for $\sigma_x^2(k)$ by the variance of a *correlated* binomial distribution [23,35]. The same calculation as in equation (2.3) gives

$$\sigma_x^2(k) = \frac{1}{k}\mu_x(k)[1-\mu_x(k)] + \frac{k-1}{k}\mathrm{Cov}(x'_i, x'_j|k). \tag{2.9}$$

Under a simplifying assumption of a bivariate normal distribution for attribute x and degree k (see appendix A),

$$\mathrm{Cov}(x'_i, x'_j|k) \approx \rho_{kx}^2 \frac{\mathrm{Var}(x)}{\mathrm{Var}(k)} \mathrm{Var}(k'|k) r_{3K}(k), \tag{2.10}$$

where $\rho_{kx} = \mathrm{Cov}(k,x)/\sqrt{\mathrm{Var}(k)\mathrm{Var}(x)}$ is the degree-attribute correlation. Then, $\sigma_x^2(k)$ is close to linear in the transsortativity value $r_{3K}(k)$, and it follows from equation (2.8) that increasing transsortativity amplifies the majority illusion. Adopting our earlier analogy, if popular people are happier, a transsortative network structure can create the perception that most people are happier, even when few people are.

We demonstrate this effect in figure 4, on power-law networks (PDF exponent $\alpha = 2.1$, degree assortativity $r_{2K} = -0.15$), with degree-attribute correlation $\rho_{kx} = 0.6$, rewired to vary mean transsortativity $\bar{r}_{3K}$ (from -0.05 to 0.4). We remove parallel edges, which are otherwise relatively common in many networks generated by the configuration model. Only 1% of the nodes have attribute value $x = 1$, while the rest have attribute $x = 0$. We show the results for $f(k)$ from an exact calculation for $k = 1, 2$ and the normal approximation in equation (2.8) for $k \geq 3$, based on the measured values of $\mu_x(k)$ and $\sigma_x(k)$. (See appendix A for details and for results on differently generated networks.) We also plot the empirically measured fraction of degree-k nodes that experience the majority illusion. In both cases, the majority illusion effect grows significantly with increasing transsortativity $\bar{r}_{3K}$: for moderate degree k, the fraction of nodes that see the 1% minority as being a majority in their neighbourhoods can be an order of magnitude larger at $\bar{r}_{3K} = 0.4$ than at $\bar{r}_{3K} = 0$. Furthermore, the model results are qualitatively consistent with the empirical results, suggesting that the tree-like approximation is justified and that degree correlations beyond transsortativity do not play an important role.

(ii) Overdispersion

While other mechanisms have been proposed for introducing correlations between neighbour attributes, their consequences are more limited. In the field of social networks, the phenomenon of *overdispersion* refers to cases where the attribute variance $\sigma_x^2(k)$ is larger than a simple binomial model would predict. This is associated with a segregation effect where nodes are unexpectedly likely or unlikely to have neighbours possessing the attribute. Empirical studies have suggested that overdispersion can occur when the neighbour attribute probability $\mu_x(k)$ itself varies from one node to another [29], and moreover that this can induce pair correlations between neighbours [22]. Indeed, from the law of total covariance, one may show (see appendix A) that $\mathrm{Cov}(x'_i, x'_j|k)$ is simply equal to the variance of the quantity $\mu_x(k)$. However, such an

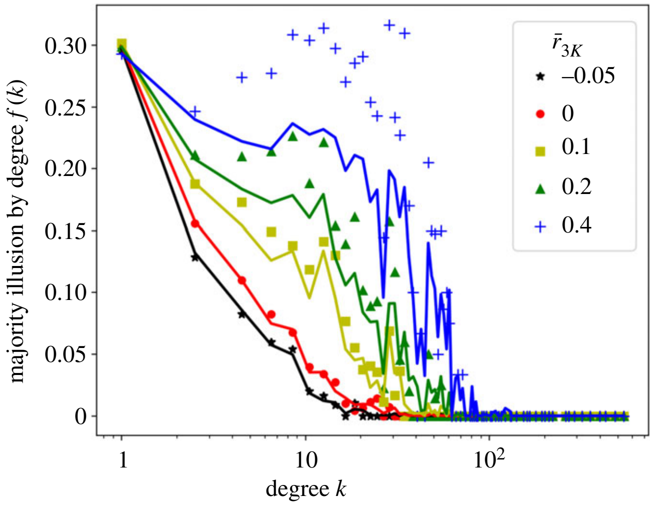

Figure 4. Transsortativity amplifies the majority illusion effect. The plot shows strength of majority illusion effect on power-law networks from configuration model with 10,000 nodes and PDF power-law exponent $\alpha = 2.1$. Networks are rewired for different mean transsortativity values, both positive and negative. 1% of nodes have binary attribute value $x = 1$, configured to create degree-attribute correlation $\rho_{kx} = 0.6$. Lines show results from binomial model (2.8) with measured mean and variance of (correlated) neighbour attribute values. Symbols show empirical fraction of degree-k nodes for which a majority of neighbours have attribute $x' = 1$. (Online version in colour.)

approach only accounts for positive neighbour correlations and a resulting increase in $\sigma_x^2(k)$ (see equation (2.9)). Transsortativity provides a mathematical framework that simultaneously includes positive and negative attribute correlations, overdispersion as well as underdispersion, and segregation of neighbour attributes.

(iii) Global cascades

Finally, we demonstrate how a network's transsortative structure can significantly alter dynamics of phenomena unfolding on it. We consider the popular Watts threshold model describing cascade dynamics [1], where nodes can be either 'active' or 'inactive'. Starting from a single active seed, nodes in the network become activated whenever more than a given fraction ϕ of their neighbours are active. This model has been used to describe contagion processes as well as the spread of ideas and opinions spread in social networks [36,37].

In the Watts model, global cascades occur when the required threshold ϕ is below a critical value ϕ^*: the largest cascade, known as the giant vulnerable cluster (GVC), then extends to a finite fraction of the network [17–19]. Figure 5 illustrates how ϕ^* varies when networks are rewired for different transsortativity values. Increasing transsortativity tends to increase the critical threshold for the GVC, from the value $\phi^* = 1/7$ predicted by the generating function formulation in [19] for $\bar{r}_{3K} = 0$, to $\phi^* = 1/2$ for $\bar{r}_{3K} = 0.3$. Just as transsortativity amplifies the majority illusion effect in low-to-moderate degree nodes, it can cause nodes to perceive a small fraction of active nodes as a large fraction of their neighbours, and become activated themselves. Thus, even moderate transsortativity can have a significant impact on the formation of global cascades.

For the special case of $\phi = 1/2$, active nodes are those experiencing the majority illusion, therefore transsortativity has the direct effect of amplifying cascade size. Figure 5 demonstrates that this effect generally occurs for large enough values of ϕ. In that regime, the cascade is sparse and spreads as a branching process, therefore the (correlated) binomial model for the majority illusion applies here. A further example is seen in figure 3, where the GVC (at $\phi = 0.2$) nearly doubles in size from the original network to the positively transsortative network. However, for

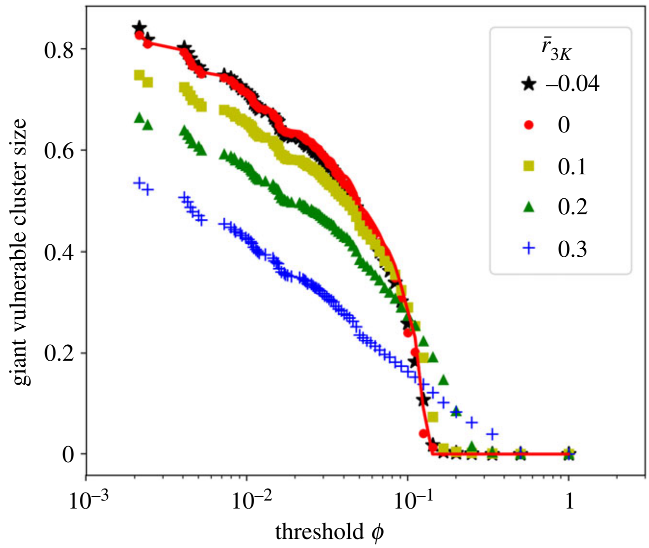

Figure 5. Transsortativity destabilizes networks to global outbreaks. The plot shows the size of cascades triggered by a single active node on power-law networks from configuration model with 10 000 nodes, PDF exponent $\alpha = 2.4$, and degree assortativity value $r_{2K} = -0.07$. Solid line shows theoretical results for baseline case $\bar{r}_{3K} = 0$: below a critical threshold value of $\phi^* = 1/7$, a finite fraction of nodes belongs to the GVC. Symbols show simulated results on networks rewired for different mean transsortativity values. (Online version in colour.)

smaller values of ϕ, cascades spread far more densely [17]. The locally tree-like approximation of the network no longer provides a valid description of neighbour activity, and figure 5 shows that increasing transsortativity suppresses rather than amplifies the GVC there. It remains an open question whether this could in part be due to a coarsening effect, where attribute segregation results in the formation of domains in the network that impede the growth of the GVC. An analogous effect has been noted in [16,18] under increasing degree assortativity.

3. Discussion

We have defined transsortativity in a network as the (two-hop) degree correlation between a pair of neighbours of a node, by analogy to degree assortativity, which represents the (one-hop) degree correlation between a node and its neighbour [16]. Transsortative structure has a significant impact on perceptions and phenomena in the network. It can significantly amplify the majority illusion effect, and increase the critical threshold for global cascades in the Watts threshold model by more than three-fold. Transsortativity partitions the network into domains where unexpectedly high or low concentrations of an attribute are observed [22,29]. In real networks, both positive and negative transsortativity occur, and we show how to increase or decrease transsortativity while preserving lower-order network statistics such as degree distribution and assortativity. Our work explains how to incorporate more realistic structure in configuration models [12,13] and degree-preserving rewiring algorithms [15] in order to better capture how real-world networks affect network phenomena.

This paper raises a number of questions to be addressed by future work. Finite size effects are known to constrain maximum degree and assortativity in scale-free networks [38], but the impact of any structural cutoff on transsortativity remains to be studied. Another interesting question is how transsortativity affects evolution of networks. It is conceivable, for example, that transsoratativity and triadic closure jointly increase assortativity in growing networks. Finally, our approach could be generalized to still higher-order structures, for example, connected subgraphs of four nodes (i.e. 4*K* structure), in cases where such expanded statistical models of networks are required.

Data accessibility. Code can be accessed in the following Git repository https://github.com/KeithBurghardt/transsortativity_code.
Authors' contributions. S.-C.N., A.P., K.B. and K.L. conceived of the research. S.-C.N. created theoretical and data analysis. S.-C.N., A.P., K.B. and K.L. discussed results and wrote the manuscript.
Competing interests. The authors declare that they have no competing interests.
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Appendix A

(a) Transsortative structure of real networks

A network's 3*K* structure is specified by degree correlations of subgraphs of three connected nodes, either 'wedges' or 'triangles', as shown in figure 6. When measuring the transsortative structure, we condition on the degree of the central node and measure the correlation of the degrees k_1 and k_2 of its neighbours.

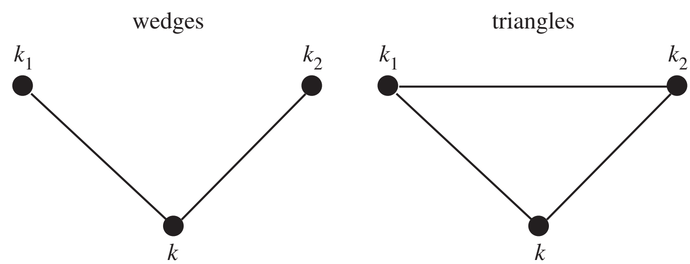

Figure 6. The two types of 3K structures: wedges and triangles.

(i) Zachary's Karate Club

We illustrate the impact of transsortativity on network structure using a widely studied benchmark, Zachary's Karate club network [32]. This network contains 34 members of a Karate club with 78 social ties between them. The degree of nodes ranges from one to 17.

We use a rewiring algorithm to change transsortativity while preserving the network's *2K* and *1K* structure. The rewiring algorithm randomly chooses two edges with two equivalent end degrees, and swaps their connections so as to change the mean transsortativity in the desired direction. The original Karate club network has a global weighted transsortativity value of −0.098. By implementing the above-mentioned algorithms, we created two network with extremely positive and negative transsortativity values of 0.4595 and −0.4021, respectively.

The original and rewired networks are shown in figure 2 of the main text. The visualization demonstrates how transsortativity can change the emergent structure of networks. When transsortativity is negative, the network appears to have a core-periphery structure, while for positive transsortativity, it appears to split into communities.

To further investigate this phenomenon, we plot $r_{3K}(k)$, transsortativity aggregated over neighbour pairs of same-degree nodes. Figure 7 shows this distribution for the original and rewired networks shown in figure 2. The rewiring procedure mainly changes the transsortativity

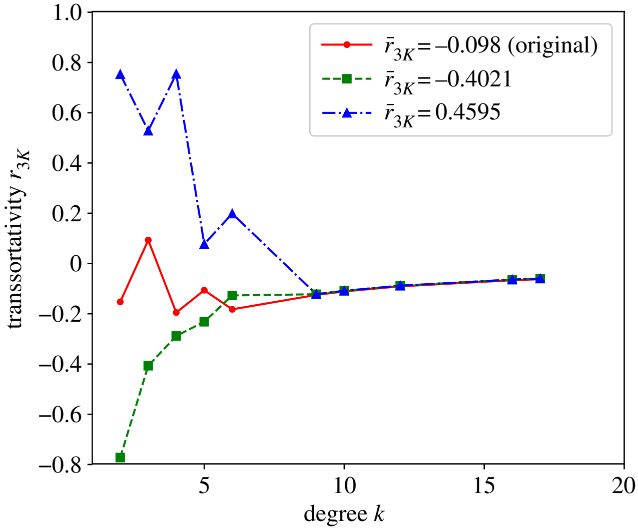

Figure 7. Contribution to mean transsortativity $\bar{r}_{3K}$ from degree specific $r_{3K}(k)$. Green squares: Zachary's Karate Club network; red circles: a rewired network of Zachary's karate club with negative transsortativity; blue triangles: a rewired network of Zachary's karate club with positive transsortativity. (Online version in colour.)

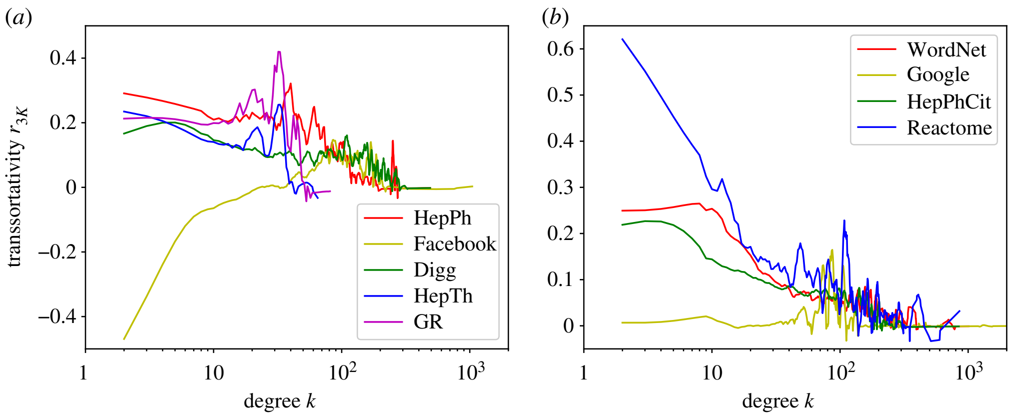

Figure 8. Contribution to mean transsortativity $\bar{r}_{3K}$ from degree specific $r_{3K}(k)$ in (*a*) social and (*b*) non-social networks. (Online version in colour.)

values of low degree nodes. For negative transsortativity, this is largely because transsortativity as a function of degree k is bounded by $-1/(k-1)$.

(ii) Other networks

Transsortativity can be substantial in real-world networks. Figure 8 shows transsortativity, $r_{3K}(k)$, aggregated over pairs of neighbours of same-degree nodes in various social, biological, technological and other networks. The real-world networks usually have significant positive values of $r_{3K}(k)$. The lower degree nodes usually have significant positive transsortativity values. This coherence with 3*K* structure is explored in the strong friendship paradox problem [23]. Transsortativity often peaks for middle degree nodes in all networks.

The six networks we study are from a variety of domains, including social networks (Facebook [31], Digg [31]), biological (Reactome [31]), Co-authorship (HepPh [31], HepTh [31])

Table 1. List of real-world networks and their basic profiles.

network	type	nodes	edges	assort. r_{2K}	clustering coeff.	transsort. $\bar{r}_{3K}$
HepPh	collaboration	12 008	118 521	0.632	0.392	0.207
Reactome	biological	6326	146 160	0.245	0.606	0.250
Digg	social	27 567	175 892	0.166	0.113	0.105
Facebook	social	4039	88 234	0.064	0.265	−0.022
HepTh	citation	34 546	420 877	−0.006	0.146	0.126
Google	techno.	875 713	4 322 051	−0.055	0.055	0.008
WordNet	semantic	146 005	656 999	−0.062	0.096	0.216
Karate club	social	34	78	−0.476	0.256	−0.098

and Semantic networks (WordNet [39]). The basic properties of networks we used in this paper are listed in table 1. In this table, we see that the number of nodes (Nodes) varies from 34 to 875 713, and the number of edges (Edges) ranges from 78 to 4 322 051. Next, we observe that the networks have a broad range of assortativity values (Assort.) that varies from −0.476 to 0.632. The mean local clustering coefficients (Cluster. Coeff.) ranges from small (0.055) to large (0.606), and finally, the mean transsortativity (Transsortativity) ranges from −0.022 to 0.250. Overall, we find more positive than negative mean transsortativity networks.

(b) Positive and negative values of transsortativity

Assume that the 2*K* structure of the network has conditional degree distribution $\nu(k'|k) = e(k,k')/q(k)$, where $e(k,k')$ is the joint degree distribution of nodes and neighbours, and $q(k)$ is the neighbour degree distribution. For simplicity, we focus our discussion on networks with three degree values: k_{H}, k_{L}, and 2. The argument can be easily extended to arbitrary degree distributions. The distribution for degree-2 nodes is

$$\nu(k_{\mathrm{H}}|2) = \nu(k_{\mathrm{L}}|2) = \frac{1}{2}. \tag{A 1}$$

The *transsortativity* is then defined as

$$r_{3K}(k) = \frac{1}{\mathrm{Var}(k'|k)} \sum_{k'_i,k'_j} k'_i k'_j \left[w(k'_i,k'_j|k) - \nu(k'_i|k)\nu(k'_j|k) \right]. \tag{A 2}$$

The joint neighbour degree distribution $w(k'_i,k'_j|k)$ denotes the probability that a degree-k node will have neighbours with degrees k'_i and k'_j. We therefore take into account how many shared neighbours of degree k each (i,j) pair has.

— *Zero transsortativity*: This happens when $w(k'_i,k'_j|2) = \nu(k'_i|2)\nu(k'_j|2)$. We can expect $w(k_{\mathrm{H}},k_{\mathrm{H}}|2) = w(k_{\mathrm{H}},k_{\mathrm{L}}|2) = w(k_{\mathrm{L}},k_{\mathrm{H}}|2) = w(k_{\mathrm{L}},k_{\mathrm{L}}|2) = \frac{1}{4}$. Namely, we can expect $\frac{1}{4}$ of degree 2 nodes are with two k_{H} neighbours, another $\frac{1}{4}$ of degree 2 nodes are with two k_{L} neighbours, and the rest $\frac{1}{2}$ of degree 2 nodes are with one k_{H} and one k_{L} neighbour. In this case, $r_{3K}(2) = 0$.
— *Positive transsortativity*: If we enforce strong positive correlation among the neighbourhood of $k = 2$ nodes without changing $\nu(k'|2)$ distribution. We can have $w(k_{\mathrm{H}},k_{\mathrm{H}}|2) = w(k_{\mathrm{L}},k_{\mathrm{L}}|2) = \frac{1}{2}$, and $w(k_{\mathrm{H}},k_{\mathrm{L}}|2) = w(k_{\mathrm{L}},k_{\mathrm{H}}|2) = 0$. Here we can expect $\frac{1}{2}$ of $k = 2$ nodes are with two k_{H} neighbours, another $\frac{1}{2}$ of degree 2 nodes are with two k_{L} neighbours. In this

case,

$$r_{3K}(2) = \left(\frac{2}{k_{\mathrm{H}} - k_{\mathrm{L}}}\right)^2 \left(\frac{1}{4}k_{\mathrm{H}}^2 - \frac{1}{2}k_{\mathrm{H}}k_{\mathrm{L}} + \frac{1}{4}k_{\mathrm{L}}^2\right) = 1. \tag{A 3}$$

— *Negative transsortativity*: If we enforce strong negative correlation among the neighbourhood of $k=2$ nodes without changing $\nu(k'|2)$ distribution. We can have $w(k_{\mathrm{H}}, k_{\mathrm{L}}|2) = w(k_{\mathrm{L}}, k_{\mathrm{H}}|2) = \frac{1}{2}$, and $w(k_{\mathrm{H}}, k_{\mathrm{H}}|2) = w(k_{\mathrm{L}}, k_{\mathrm{L}}|2) = 0$. Here we can expect all of $k=2$ nodes are with one k_{H} and one k_{L} neighbour. In this case,

$$r_{3K}(2) = \left(\frac{2}{k_{\mathrm{H}} - k_{\mathrm{L}}}\right)^2 \left(-\frac{1}{4}k_{\mathrm{H}}^2 + \frac{1}{2}k_{\mathrm{H}}k_{\mathrm{L}} - \frac{1}{4}k_{\mathrm{L}}^2\right) = -1. \tag{A 4}$$

(c) Effect of transsortativity on majority illusion

(i) Relation between degree assortativity and 2K attribute correlations

In this section, we consider aspects of a network's 2K structure, and in particular, how correlations between degrees of connected nodes (degree assortativity) relate to correlations between the attributes of connected nodes (homophily). If $e(k,k')$ represents the joint distribution of pairs of adjacent nodes, then the covariance of the attributes of two adjacent nodes x and x' is

$$\begin{aligned}\mathrm{Cov}(x,x') &= \sum_{k,k'} P(x=1|k)e(k,k')P(x'=1|k') - \langle x\rangle^2 \\ &= \sum_{k,k'} E(x|k)e(k,k')E(x'|k') - \langle x\rangle^2.\end{aligned} \tag{A 5}$$

Let us adopt the simplifying assumption that attribute x and degree k are described by a bivariate normal distribution. In that case, a standard result on conditional expectation gives

$$E(x|k) = \langle x\rangle + \rho_{kx}\frac{\sigma_x}{\sigma_k}(k - \langle k\rangle), \tag{A 6}$$

where ρ_{kx} is the degree–attribute correlation, σ_x^2 is the variance of the attribute distribution, and σ_k^2 is the variance of the network degree distribution $p(k)$. Then

$$\begin{aligned}\mathrm{Cov}(x,x') &= \sum_{k,k'}\left[\langle x\rangle + \rho_{kx}\frac{\sigma_x}{\sigma_k}(k-\langle k\rangle)\right]\left[\langle x\rangle + \rho_{kx}\frac{\sigma_x}{\sigma_k}(k'-\langle k\rangle)\right]e(k,k') - \langle x\rangle^2 \\ &= \langle x\rangle \sum_{k,k'}\rho_{kx}\frac{\sigma_x}{\sigma_k}(k+k'-2\langle k\rangle)e(k,k') + \rho_{kx}^2\frac{\sigma_x^2}{\sigma_k^2}\sum_{k,k'}(k-\langle k\rangle)(k'-\langle k\rangle)e(k,k') \\ &= \langle x\rangle\rho_{kx}\frac{\sigma_x}{\sigma_k}\left(2\frac{\langle k^2\rangle}{\langle k\rangle} - 2\langle k\rangle\right) + \rho_{kx}^2\frac{\sigma_x^2}{\sigma_k^2}\left(r_{2K} + \frac{\langle k^2\rangle^2}{\langle k\rangle^2} - 2\langle k^2\rangle + \langle k\rangle^2\right) \\ &= 2\langle x\rangle\rho_{kx}\frac{\sigma_x}{\sigma_k}\frac{\sigma_k^2}{\langle k\rangle} + \rho_{kx}^2\frac{\sigma_x^2}{\sigma_k^2}\left(r_{2K}\sigma_k^2 + \frac{\sigma_k^4}{\langle k\rangle^2}\right) \\ &= 2\rho_{kx}\sigma_x\sigma_k\frac{\langle x\rangle}{\langle k\rangle} + \rho_{kx}^2\sigma_x^2\sigma_k^2\left(r_{2K} + \frac{\sigma_k^2}{\langle k\rangle^2}\right).\end{aligned} \tag{A 7}$$

(ii) Relationship between transsortativity and 3K neighbour attribute correlations

In this section, we consider aspects of a network's 3K structure, and in particular, how correlations between the degrees of pairs of neighbours (transsortativity) relate to correlations between the attributes of pairs of neighbours. Consider a node, two of whose neighbours have degrees k_i', k_j'

and attributes x'_i, x'_j. Then the correlation between the attributes is

$$\begin{aligned}
\mathrm{Cov}(x'_i, x'_j|k) &= \sum_{x'_i=0,1}\sum_{x'_j=0,1} x'_i x'_j P(x'_i, x'_j|k) - \left[\sum_{x'=0,1} x' P(x'|k)\right]^2 \\
&= P(x'_i = 1, x'_j = 1|k) - \left[P(x'=1|k)\right]^2 \\
&= \sum_{k'_i,k'_j} P(x'_i = 1|k'_i) w(k'_i, k'_j|k) P(x'_j = 1|k'_j) - \left[\sum_{k'} P(x'=1|k')\nu(k'|k)\right]^2 \\
&= \sum_{k'_i,k'_j} E(x'_i|k'_i) w(k'_i, k'_j|k) E(x'_j|k'_j) - \left[\sum_{k'} E(x'|k')\nu(k'|k)\right]^2.
\end{aligned} \tag{A 8}$$

Again using the simplifying assumption (equation (A 6)) that attribute x and degree k are described by a bivariate normal distribution

$$\begin{aligned}
\mathrm{Cov}(x'_i, x'_j|k) &= \sum_{k'_i,k'_j}\left[\langle x\rangle + \rho_{kx}\frac{\sigma_x}{\sigma_k}(k'_i - \langle k\rangle)\right]\left[\langle x\rangle + \rho_{kx}\frac{\sigma_x}{\sigma_k}(k'_j - \langle k\rangle)\right] w(k'_i, k'_j|k) \\
&\quad - \left(\sum_{k'}\left[\langle x\rangle + \rho_{kx}\frac{\sigma_x}{\sigma_k}(k' - \langle k\rangle)\right]\nu(k'|k)\right)^2 \\
&= \langle x\rangle^2 + \langle x\rangle \sum_{k'_i,k'_j}\rho_{kx}\frac{\sigma_x}{\sigma_k}(k'_i + k'_j - 2\langle k\rangle) w(k'_i, k'_j|k) \\
&\quad + \rho_{kx}^2\frac{\sigma_x^2}{\sigma_k^2}\sum_{k'_i,k'_j}(k'_i - \langle k\rangle)(k'_j - \langle k\rangle) w(k'_i, k'_j|k) \\
&\quad - \left(\langle x\rangle + \sum_{k'}\rho_{kx}\frac{\sigma_x}{\sigma_k}(k' - \langle k\rangle)\nu(k'|k)\right)^2 \\
&= \langle x\rangle^2 + 2\rho_{kx}\frac{\sigma_x}{\sigma_k}\langle x\rangle\left[E(k'|k) - \langle k\rangle\right] \\
&\quad + \rho_{kx}^2\frac{\sigma_x^2}{\sigma_k^2}\left[r_{3K}(k)\mathrm{Var}(k'|k) + E(k'|k)^2 - 2E(k'|k)\langle k\rangle + \langle k\rangle^2\right] \\
&\quad - \left[\langle x\rangle + \rho_{kx}\frac{\sigma_x}{\sigma_k}\left[E(k'|k) - \langle k\rangle\right]\right]^2 \\
&= \rho_{kx}^2\frac{\sigma_x^2}{\sigma_k^2}\mathrm{Var}(k'|k) r_{3K}(k).
\end{aligned} \tag{A 9}$$

Thus, for positive ρ_{kx}, increasing transsortativity $r_{3K}(k)$ amplifies attribute correlations between pairs of neighbours.

(iii) Neighbour attribute correlations as a consequence of nonuniform $\mu_x(k)$

In a binomial model where neighbour attributes are specified by $P(x'_i = 1|k) = \mu_x(k)$, independently for all neighbours of a node, there can still be correlations between neighbour attribute values when the quantity $\mu_x(k)$ is itself unknown and drawn randomly. This is a consequence of the law of total covariance.

As an analogy, imagine that each neighbour's attribute was decided from a coin flip, but the coin has an unknown bias b so that $P(x'_i = 1|k) = b$. The law of total covariance states that

$$\mathrm{Cov}(x'_i, x'_j|k) = E(\mathrm{Cov}(x'_i, x'_j|k, b)) + \mathrm{Cov}(E(x'_i|k, b), E(x'_j|k, b)). \tag{A 10}$$

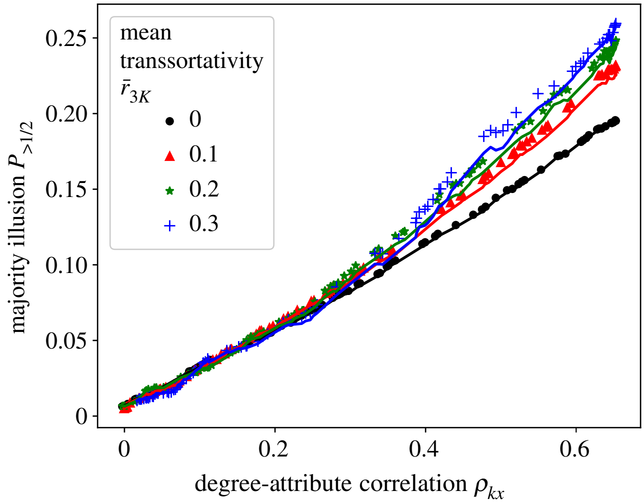

Figure 9. Strength of majority illusion effect on power-law networks. Symbols show fraction of nodes for which a majority of neighbours have attribute $x' = 1$. Lines show theory from equations (2.7) and (2.8), given measured mean and variance of neighbour attribute values, and exact values calculated for $k = 1$ and 2 (see 'Numerical analysis'). Networks are a configuration model network with 10 000 nodes and PDF power-law exponent $\alpha = 2.4$ rewired to reach different mean transsortativity values. 1% of nodes have binary attribute value $x = 1$, assigned according to specific degree-attribute correlation values ρ_{kx}. (Online version in colour.)

By construction, the covariance of $x'_i|k$ and $x'_j|k$ is zero *conditional on b*, so

$$\begin{aligned}\mathrm{Cov}(x'_i, x'_j|k) &= \mathrm{Cov}(E(x'_i|k, b), E(x'_j|k, b))\\ &= \mathrm{Cov}(P(x'_i = 1|k, b), P(x'_j = 1|k, b))\\ &= \mathrm{Cov}(b, b)\\ &= \mathrm{Var}(b).\end{aligned} \tag{A 11}$$

Thus, if $P(x'_i = 1|k) = \mu_x(k)$ but $\mu_x(k)$ itself varies from one node to another,

$$\mathrm{Cov}(x'_i, x'_j|k) = \mathrm{Var}(\mu_x(k)). \tag{A 12}$$

Previous studies in the social network literature [22,29] have used precisely the mechanism of varying attribute propensity to create overdispersion (monophily) of neighbour attributes. The expression above replicates the empirical observation by Altenburger & Ugander [22] that this induces attribute similarities between friends-of-friends (x'_i and x'_j) without necessarily requiring similarity among friends (x' and x). However, note that since $\mathrm{Var}(\mu_x(k))$ is nonnegative, this mechanism can only account for positive neighbour correlations, unlike transsortativity which can give rise to both positive and negative values of $\mathrm{Cov}(x'_i, x'_j|k)$.

(iv) Numerical analysis

From the 2*K* and 3*K* structures of the network, we can measure the values of $\mu_x(k)$ and $\sigma_x^2(k)$ for each value of k. If the attributes of the nodes are restricted to be binary, the information is sufficient to give the correct strength of the majority illusion for low degree classes. For degree $k = 1$, as have $f(1) = \mu_x(1)$, as it only has a single neighbour. For degree $k = 2$, a node observes the majority illusion only when both neighbours bear $x' = 1$, or $f(2) = p_{11}$. Considering the three possible cases of neighbour pairs (p_{11}, p_{10}, p_{00}), we can write the relations with $\mu_x(k)$ and $\sigma_x^2(k)$ by the first and the second moment

$$\mu_x(k) = p_{11} + \frac{1}{2}p_{10} \quad \text{and} \quad \sigma_x^2(k) = p_{11} + \frac{1}{4}p_{10} - [\mu_x(k)]^2 . \tag{A 13}$$

The two moments do not depend on p_{00}. We can solve this system of two equation and two unknowns to get $f(2) = p_{11} = 2\sigma_x^2(k) + 2\mu_x^2(k) - \mu_x(k)$.

For higher values of k, we use the Gaussian approximation of $f(k)$ to calculate the strength of the majority illusion with correlated neighbours. We define the 'majority illusion' as nodes observing *more than* 1/2 of their neighbours in the active state. Therefore, we need to move the lower bound in the integral of the Gaussian PDF to exclude the cases in which nodes see 1/2 or fewer of their neighbours in the active state. The lower bound is $\frac{1}{2}$ for odd degree classes, but $\frac{1}{2} + \frac{1}{2k}$ for even degree classes. By using the procedure above, we produce the lines plotted in figure 3 in the main paper, for networks of varying transsortativity generated from power-law degree distributions with PDF exponent $\alpha = 2.1$. Those results show that the locally tree-like approximation and the normal approximation to the binomial distribution are adequate to describe the majority illusion. Note that the latter approximation further assumes that moments higher than the variance can be neglected, and so there are no higher-order correlations beyond 3K, such as those involving connected subgraphs of four nodes.

We also display similar results in figure 9, for power-law networks with exponent $\alpha = 2.4$.

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